**Diffraction & Interference**

**Introduction**

In 1704, Sir Isaac Newton postulated a theory that light is made up of particles. After all, a picture of light as a stream of particles readily explains the apparent fact that light travels in straight lines. Because of Newton’s enormous reputation this theory remained dominant well into the 19th century.

This theory would be challenged in 1801 by the experimental demonstration that light can undergo interference and diffraction - both wave-like properties. Performed by Thomas Young, (the same Thomas Young who deciphered the Rosetta stone), this experiment was very important because it showed that light also exhibited wave-like properties.

In today’s exercise, you will perform experiments that illustrate the wave-like nature of light.

**Equipment**

- Helium-Neon Laser
- Cornell Slit Film
- Spectral Tube Lamp
- Various Gas Tubes
- Diffraction Grating

**Background**

Diffraction is the bending of a wave around an obstacle, while interference is the result of the superposition (combining the amplitudes) of two or more over-lapping waves. These effects occur in all wave types, whether they consist of water, sound, or light. Because the wavelength of light is so short (the visible light range is approximately 400-700 nm), the wave properties of light can be overlooked.

Modern quantum theory holds that light has both wave-like and particle-like properties. When the length scales involved are large compared to the wavelengths of light (ex., forming images with thin lenses), the particle nature of light dominates. On the other hand, when the length scales are comparable to or smaller than the wavelengths of light, the wave nature of light dominates. In your experiments today, you can safely ignore the particle properties of light and treat it purely as a wave because the diffraction gratings through which you will either pass or reflect light have dimensions comparable to that of the wavelength of the light source.

Huygen’s Principle states that when a plane wave falls perpendicularly on a pair of slits ($S_1$ & $S_2$), each slit becomes a secondary source of light having the same wavelength ($\lambda$), frequency ($f$), and phase ($\delta$). However, to reach some point $P$ on a distant screen, light from slit 2 travels farther than light from slit 1 and arrives with a different phase (see Figure 1).

The phase difference $\delta$ (in radians) equals $2\pi x$ (extra distance)/$\lambda$ where “extra distance” is the additional path traveled by light emanating from the slit farthest away from point P compared to light emanating from the closer slit. When the light from the two slits combines at $P$, the amplitude of the resulting wave depends on the phase difference. If $\delta = 0, 2\pi, 4\pi$, etc., the waves are in phase and their amplitudes add giving a maximum intensity - they interfere **constructively**. If $\delta = \pi, 3\pi, 5\pi$, etc., the amplitudes subtract giving a minimum intensity – this is **completely destructive interference**. For other phase differences, the resulting interference is partially destructive, and the resulting intensity at $P$ will have a value somewhere between the minimum and maximum values.

In many cases, the distance to $P$ is so large compared to the slit separation ($L >> d$) that the light paths from the two slits to $P$ are almost parallel, and the path length difference is simply $d\sin(\theta)$ where $d$ is the separation of the slits and the angle $\theta$ is shown in Figure 1. The brightest points on the screen (the interference maxima) occur at angles given by the equation $d\sin(\theta) = m\lambda$, where $d$ is the distance separating the slits, and $m$ is an integer indicating the order of the interference maximum ($m = 0, \pm 1, \pm 2, \pm 3, ...$).
Slit-Film Diffraction & Interference

In the first part of this lab, you will study the diffraction and interference of light using a helium-neon laser and a slitfilm, which is a piece of opaque material pressed between two pieces of glass. This opaque material has narrow lines or slits cut into it, which allow light to pass through the film. When you look closely through the slitfilm you can easily see the different patterns. There are several different patterns of parallel slits in the film that allow you to perform single-, double- or multiple-slit diffraction experiments.

Laser Diffraction

The laser is an important tool in science because it is a source of coherent light, meaning that nearly all of the light coming from a laser has the same frequency, the same wavelength, the same phase and nearly the same direction. For this reason, laser light can be treated as an ideal plane wave.

Warning: The low-power helium-neon gas laser beam used in these experiments will not cause permanent damage to your retina, but it can produce annoying after-images that may persist for several minutes or longer. DO NOT allow the beam to shine (either directly or by bouncing off a shiny surface) into any one’s eyes.
**Procedure**

- Use Figure 3 as a guide to construct the experimental display for diffraction studies. Use the Cornell slitfilm (the 8 x 10 cm. glass plate) mounted in front of the laser; the mounting allows for positioning the plate in the x and y directions. The diagram in Appendix 1 shows the location and separations of all the slit configurations. The separation between slits, \( d \), is printed below each pattern. The units are millimeters.
- Shine the laser through the 30-slit grating. An interference pattern will be produced on a segment of meter stick used as a screen a distance, \( L \), away.
- Measure the distance \( x_1 \) on the screen from the zeroth (0th) order maximum to the 1st order maximum on the right.
- Repeat the measurement for the 1st order maximum on the left.
- Find an average for your two distances.
- Calculate the angle \( \theta_1 \) and the wavelength \( \lambda \) of the laser; the wavelength of red laser light is approximately 600-650 nanometers. If your result is substantially different, re-check your procedures.

![Figure 3: Experimental set-up for diffraction experiments.](image)

Now use the 80-slit grating to obtain a more accurate value for the wavelength.

- As in the last part, measure the average distance \( x_m \) for several orders \((m = 1, 2, 3)\) of maxima and calculate the angle \( \theta_m \) for each.
- Use Excel to produce a plot of \( \sin(\theta_m) \) against the order number \( m \). Configure the plot with a linear fit, and from its slope determine the wavelength \( \lambda \) of the laser light.
Finding the Spacing of Slits in a Diffraction Grating

The second device you will use is called a diffraction grating, which consists of a piece of plastic that has had a series of parallel grooves (more than 10,000 per inch) pressed into it. The wavelength of the laser light that you have just found can be used to accurately determine the spacing between the lines.

- Slide the Cornell slitfilm plate out of the way and place the diffraction grating stand on the small shelf.
- The remaining procedure is almost the same as for finding the wavelength in the previous section, except that in this case, you will calculate $d$ while already knowing the wavelength, $\lambda$, which is the same value as found above.
- **Important Tip:** Since the spacing $d$ is very small in the diffraction grating, the angles $\theta_m$ of the diffracted spots will be much, much larger than when using the 80-mm slitfilm. You will need to make the distance $L$ to the screen much, much smaller in this case so you can see a number of the maxima on the meter stick.

Spectral Lines

When a high voltage is applied across a gas-filled tube, electrons, accelerated by the high voltage, collide with the gas atoms. In these collisions, the atoms absorb energy and become internally excited. The excited atoms subsequently give up this extra energy in the form of light, which is how a neon sign works.

The most striking thing about the light that is emitted from the excited gas is that its spectrum contains only certain wavelengths. The spectral lines that are emitted by a particular gas are characteristic of that gas; no two gases (element or compound) have identical spectral patterns.

In this part of the experiment, you will use the diffraction grating to examine the spectrum of a gas excited by an electrical discharge. Each spectrum consists of sharp, well-defined lines of a particular color. Since color is an indicator of wavelength, each color that you observe represents energy (in the form of photons) of a particular wavelength being emitted by the gas. As a consequence, each color will be diffracted at a different angle, which is the reason that you see them as separated lines. The existence of discrete lines (specific wavelengths) instead of a continuous spectrum (like that of an incandescent light bulb) was important in the discovery of quantum physics.

First, investigate the properties of a continuous spectrum:

- Observe the spectrum emitted by an ordinary tungsten light bulb by holding the diffraction grating close to your eye.
- You should observe that the spectrum appears continuous - there seem to be no gaps in the spectrum as the colors blend into one another. This is because this type of light source produces nearly every wavelength of light in the visible spectrum, so the discrete lines that are present are too close to be distinguished from one another.

Set up the experimental display shown in Figure 4 to observe light from a non-continuous spectrum source: a hydrogen gas lamp. The light emitted from the tube will again be diffracted by the grating. This time, instead of a whole, continuous spectrum, you should see just a few distinct lines of color.

- Place the diffraction grating at a known distance $L$ from the hydrogen gas lamp ($L$ should be ~ 1 meter); see Figure 4.
- Observe that the hydrogen gas emits only a few certain wavelengths. What colors do you observe?
- As you would expect from your previous observations through the diffraction grating, the spots are repeated on both sides of the center and at succeeding distances, ($m = 1, 2, 3, \ldots$); is this what you observe? If not, explain why this might be the case.
- To measure the angle $\theta$ of the 1st interference maximum of each color in the spectrum, look at the tube through the diffraction grating, and direct your lab partner to mark the location of the color line with a straw mounted in a wooden block.
Find the angle $\theta$ for each of the spectral lines. Calculate the wavelengths of each of the colors that you observe for the hydrogen gas, using $m\lambda = d \sin(\theta)$, which simplifies to $\lambda = d \sin(\theta)$, because we are studying the first order pattern ($m = 1$).

Balmer Series

The visible lines of the spectrum of atomic hydrogen are called Balmer lines in honor of Johann Balmer, a Swiss schoolteacher who discovered in 1884 an empirical formula that predicted the observed wavelengths.

This is the Balmer formula:

$$\frac{1}{\lambda} = \frac{1}{91.127} \left( \frac{1}{2^2} - \frac{1}{p^2} \right)$$

where $p = 3, 4, 5, \ldots$.

Balmer also conjectured that his formula could be generalized to predict the placement of the other lines of the hydrogen spectrum in the infrared and the ultraviolet regions of the electromagnetic spectrum. This was indeed found to be the case, and so the generalized Balmer formula is given by:

$$\frac{1}{\lambda} = \frac{1}{91.127} \left( \frac{1}{n^2} - \frac{1}{p^2} \right)$$

where $n = 1, 2, 3, \ldots$, and $p = (n+1), (n+2), (n+3), \ldots$
Thirty years passed before anyone was able to derive Balmer’s formula as a consequence of a theory of the structure of the hydrogen atom. In 1913, Bohr successfully introduced quantum ideas and showed that the hydrogen atom could exist only in certain allowed energy states given by:

\[ E_n = -\frac{R\hbar}{n^2} = -\frac{13.6}{n^2}\text{eV} \]

\[ R = \frac{mk^2e^4}{4\pi\hbar^3} = \frac{1}{91.127}\text{(nm)}^{-1} \]

where

- \( k \) is Coulomb’s constant
- \( e \) is the electron charge
- \( m_e \) is the electron mass
- \( \hbar \) is Planck’s constant \((\hbar / 2\pi)\)
- \( n = 1, 2, 3, \ldots \) is the principal quantum number which specifies the allowed energy levels.

A transition down from level \( p \) to level \( n \) involves an energy decrease of \( \Delta E \):

\[ \Delta E = E_p - E_n = R\hbar\cdot\left[\frac{1}{n^2} - \frac{1}{p^2}\right] \]

This released energy is carried off by a photon. According to Einstein, a photon’s energy is \( hf \), with \( f \) the frequency of the photon and \( h \) Planck’s constant. So we have:

\[ f = \frac{E}{h} = R\cdot\left[\frac{1}{n^2} - \frac{1}{p^2}\right] \]

All that remains is to recall that the wavelength \( \lambda \), frequency \( f \), and the speed of light \( c \) are related by \( f = c/\lambda \). The Bohr theory says that the wavelength of the light emitted by the atom is:

\[ \frac{1}{\lambda} = R\cdot\left[\frac{1}{n^2} - \frac{1}{p^2}\right] \]

which is identical to Balmer’s generalized formula.

- Use the Balmer series to compute the theoretical wavelengths for the visible portion of hydrogen’s spectrum. Compare these predictions with your experimentally measured values.

The spectra of other elements and compounds can be measured using the same method as for hydrogen. Quantum mechanics can predict the wavelengths of atoms other than hydrogen with great accuracy, but the calculations are quite involved. Simple formulae apply only for single electron atoms such as H, He+, Li++, etc. Unfortunately, there are no formulae (like the Balmer formula) that enable you to easily calculate these wavelengths.

- Carefully observe the visible spectrum for the other gases that have been provided (neon, helium, mercury vapor).
- Use the chart on the wall to determine which spectra correspond to which elements.
Concluding Questions

When responding to the questions/exercises below, your responses need to be complete and coherent. Full credit will only be awarded for correct answers that are accompanied by an explanation and/or justification. Include enough of the question/exercise in your response that it is clear to your teaching assistant to which problem you are responding.

Use the information below and the picture in Figure 5 for the first 3 exercises:

Red light from a distant point source is incident upon two identical, very narrow slits. Figure 5 shows the interference pattern that appears on a distant screen, as well as a basic schematic of the experimental apparatus. Note that point C is at the center of the screen.

1. How does this pattern differ from what you would have predicted if you had used the idea that light travels in straight lines through the slits.

2. For each of the lettered points in the photograph above, determine the path difference ($Δx$) in terms of the wavelength ($λ$) and the phase difference ($Δϕ$) between the waves emanating from the two slits.

3. Suppose that a single change was made to the double slit experiment described above. For each case below refer to the interference pattern shown above, and determine how, if at all, that the change would affect the resulting interference pattern on the screen. Explain your reasoning.
   (a) The distance between the slits is decreased (without changing the width of the slits).
   (b) The screen is moved closer to the two slits emitting the light.
   (c) The red light source is replaced with a green light source.

4. If you looked at an incandescent light bulb and a neon sign through a diffraction grating, describe the differences you would observe in the spectra. Explain your reasoning.
Appendix 1: Cornell Slitfilm

Use the center-to-center slit separation (printed below each pattern, in mm) as the quantity \( d \) in the formula above.

The three numbers listed vertically by each configuration of slits show:
- \( n \) = number of slits
- \( w^1 \) = width of the slit
- \( s^1 \) = width of opaque space between slits

\[1^\text{The variables w and s have units of 0.04393 mm}\]
Appendix 2: Multiple Slits

The introductory remarks described the way in which two slits give rise to an interference pattern, but the films that you use have many slits, all with the same separation $d$. You have to add in the extra waves at $P$ from these extra slits, taking proper account of their phase shifts. Each new slit will add in a wave shifted in phase by $\delta$ from the one before.

However, the condition for constructive interference of the light from all the slits is unchanged - the interference maxima remain at the same angles as would be the case for two slits. The main difference is that the maxima become narrower and narrower as the number of slits increases.

For $N$ slits, the angular width of the maxima ($\theta_n$) is given by:

$$d \sin(\theta_n) = \frac{\lambda}{N}$$

Narrow maxima are just what is required to distinguish two almost equal wavelengths. This is why diffraction gratings with $N = 10000$ or more are so useful in studying atomic and molecular spectra.