Physics 318
Homework 12

1. This problem can be solved by using the Dirac Equation in $x$-space or by using the Dirac Equation in $p$-space. In $x$-space the equation is $\gamma^\mu p_\mu \psi = mc\psi$ where $\psi = NE^{-ip_\mu x^\mu/\hbar}u$. In $x$-space the $p$ operators act as derivatives and the $x$ operators act as multipliers. By plugging in $u^{(1)} = \begin{pmatrix} 1 \\ 0 \\ cp/(E + mc^2) \\ 0 \end{pmatrix}$ into the equation you see that $u^{(1)}$ is in fact a solution. In $p$-space the equation is $\gamma^\mu p_\mu u(p) = mcu(p)$ with the $p$ operators acting as multipliers and the $x$ operators acting as derivatives. Plugging in the same $u^{(1)}$ you again see that this is a solution to the Dirac Equation.

2. First of all $\vec{B} = \vec{\nabla} \times \vec{A} = B\hat{z}$ which checks out.

   (a) $[\Pi_x, \Pi_y] = -i\hbar B e/c$

   (b) Following the hint given in class let $\alpha = -B e/c$ and $Y = \Pi_y$ which yields $[Y, \Pi_y] = i\hbar$. The Hamiltonian given in class can be written as $H = \frac{1}{2m}(\Pi_x^2 + \Pi_y^2 + p_z^2) = \frac{1}{2m}(\alpha^2 Y^2 + \Pi_y^2 + p_z^2)$. The last term in this Hamiltonian gives the $\frac{k^2}{2m}$ contribution to the eigenvalues. The remaining contribution comes from comparing the first 2 terms in this those of the Hamiltonian of the harmonic oscillator ($H = \frac{1}{2}m\omega^2 x^2 + \frac{p_z^2}{2m}$). You see that $\frac{\alpha^2}{2m}$ corresponds to $\frac{m\omega^2}{2}$ meaning that $\omega$ corresponds to $|\alpha/m|$. When you make this substitution you see that $E_{k,n} = \frac{k^2}{2m} + \hbar |\alpha/m|(n + \frac{1}{2}) = \frac{k^2}{2m} + \frac{|eB|\hbar}{mc}(n + \frac{1}{2})$.

3. See diagrams below.

4. (a) Conservation of charge is violated.
   (b) Baryon number is not conserved.
   (c) Energy is not conserved.
   (d) Conservation of Lepton number is violated