Physics 217

Homework 6

1. See Figure 1. \( \langle \psi_1 | \hat{x} | \psi_1 \rangle = \frac{2}{a} J_{-a/2}^{a/2} x \cos^2(\frac{n\pi x}{a})dx = 0 \) because you are integrating an odd function over even limits, likewise \( \langle \psi_2 | \hat{x} | \psi_2 \rangle = 0 \).

2. (a) From Figure 2 you can see that you are most likely to find the particle in the \( x \) interval \([0,50]\).
   
   (b) Doing the relevant integration you see that the wavefunction is normalized. Figure 3 shows the expectation value of \( x \). This makes sense because the largest amount of the wavefunction was located in this interval.

3. (a) \( \omega_1 = \frac{E_1}{\hbar} = \frac{\pi^2 \hbar}{2ma^2} \) and likewise \( \omega_2 = \frac{\pi^2 \hbar}{ma^2} \). \( \psi_1 \) evolves as \( \exp[-i\frac{E_1 t}{\hbar}] = \exp[-i\omega_1 t] \) and \( \psi_2 \) evolves as \( \exp[-i\omega_2 t] \).
   
   (b) Figure 4 you can see that the \( \text{wavefn}(x,t) \) returns the same thing as \( \text{wavefn}(0)(x) \).
   
   (c) In the animated plot the probability density oscillates between the positive and negative \( x \) regions with a period of \( \sim 42 \).
   
   (d) Figure 5 shows the plot of \( \text{expected}_x(t) \).

4. \( \langle \Psi | x | \Psi \rangle = J_{-a/2}^{a/2} \Psi^* x \Psi dx \). Using the result of question one we know that the terms which look like \( \psi_1^* x \psi_1 \) and \( \psi_2^* x \psi_2 \) don’t contribute anything. Thus we are left with \( \frac{1}{a} \left( e^{i\frac{\Delta E t}{\hbar}} + e^{-i\frac{\Delta E t}{\hbar}} \right) J_{-a/2}^{a/2} x \cos(\frac{n\pi x}{a}) \sin(\frac{2\pi x}{a}) dx \). Upon substitution and making use of the given integral and Euler’s equation we get \( \langle x \rangle = \frac{16a}{9\pi^2} \cos(\frac{\Delta E t}{\hbar}) \). Plugging the appropriate values we see that the amplitude and period match the earlier value \( (\text{Amp} \sim 18.01 \text{ and } T \sim 42) \).
Figure 1: $\psi_1$ and $\psi_2$

Figure 2: $\psi(x)$ and $|\psi(x)|^2$

Figure 3: The expectation value of $x$. 

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Figure 4: Initial wavefunction using new .m file.

Figure 5: plot of expected $x(t)$