Physics 217
Homework 3

1. (a) 4.12: Angular momentum is given by the equation \( \vec{L} = m\vec{\omega} \times \vec{r} \) which yields units of K.g \( \cdot \) m \( \cdot \) s. Planck’s constant has units of J \( \cdot \) s = K.g \( \cdot \) m \( ^2 \) \( \cdot \) s \( ^2 \) \( \cdot \) s = K.g \( \cdot \) m \( \cdot \) s \( \cdot \) m.

(b) 4.14: \( F_g = \frac{GM_pM_e}{a^2} = 3.62 \times 10^{-47} \) N, \( F_C = \frac{e^2}{4\pi\epsilon_0 a^2} = 8.24 \times 10^{-8} \) N. You can see that \( \frac{F_C}{F_g} \approx 2 \times 10^{39} \) so it is justifiable to ignore the force of gravity here.

2. The magnetic dipole moment is given by the equation \( \mu = IA = I\pi a^2 \) where \( I \) is the current passing around an area \( A \) and \( a \) is the Bohr radius. The electron orbits the nucleus in a time of \( \frac{8\pi^2a}{Ze^2/n\hbar} \) yielding a current of \( I = \frac{mZ^2e^5}{32\pi^3\epsilon_0n^3\hbar^2} \). This combination gives \( \mu = \frac{\epsilon_0 a}{2m} n\hbar \). It is known that the angular momentum is quantized by the equation \( L = n\hbar \), thus \( \frac{\mu}{L} = \frac{\epsilon_0 a}{2m} \) which is independent of \( n \) (the number of the Bohr orbit).
3. Above is a picture of the coordinate system which was used in assigning the vector properties.
   (a) \( n=1 \)
   (b) \( r = a_0 = 5.29 \times 10^{-11} \text{m} \)
   (c) \( \vec{L} = m\vec{v} \times \vec{r} = n\hbar \hat{z} = \hbar \hat{z} \)
   (d) \( \vec{p} = m\vec{v} = 2 \times 10^{-24} \frac{\text{kg} \cdot \text{m}}{\text{s}} \hat{x} \)
   (e) \( \omega = v/a_0 = 4.15 \times 10^{16} \text{s}^{-1} \)
   (f) \( v = 2.2 \times 10^6 \text{m/sec} \)
   (g) As you observed in problem 1 the force can be approximated to be the Coulomb force. \( F_C = 8.24 \times 10^{-8} \text{N} \).
   (h) \( a = v^2/a_0 = 9.1 \times 10^{22} \text{m/s}^2 \)
   (i) \( KE = \frac{1}{2}mv^2 = 2.2 \times 10^{-18} \text{J} = 13.7 \text{eV} \)
   (j) \( V = -\frac{e^2}{4\pi\varepsilon_0 a_0} = -4.4 \times 10^{-18} \text{J} = -27.5 \text{eV} \)
   (k) \( E = KE + V = -2.2 \times 10^{-18} \text{J} = -13.8 \text{eV} \)

According to equation 4-16 the radius of the orbit varies as \( n^2 \). The total energy is a combination of the kinetic and potential energies which both vary as \( n^{-2} \) thus \( E \propto n^{-2} \).

4. (a) Let the initial energy of the system be \( E_0 \) (the atom is the only thing which contains energy). After emission of the photon the atom has energy \( E_f \), the photon
has energy $h\nu$, and the atom has ‘recoil’ energy $\frac{p^2}{2m}$. Equating initial and final energy and using $p = h\nu/c$ you arrive at $\frac{h^2}{2mc^2}v^2 + h\nu - \Delta E = 0$. Making use of the quadratic equation you get $\nu = \frac{mc^2}{h} \left[ -1 \pm \left( 1 + 2\frac{\Delta E}{mc^2} \right)^{1/2} \right]$. Next you must use the expansion of the binomial series which is $(1 + x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2$ for small $x$. This yields $\nu = \frac{mc^2}{h} \left[ -1 \pm \left[ 1 + \frac{\Delta E}{mc^2} - \frac{1}{2} \left( \frac{\Delta E}{mc^2} \right)^2 \right] \right]$. Taking the positive root (if you take the negative root you would arrive at a negative frequency and what sense does that make) you arrive at $\nu = \frac{\Delta E}{h} \left[ 1 - \frac{\Delta E}{2mc^2} \right]$ where the term in brackets is the desired correction.

(b) The second term in the correction is very small $\frac{\Delta E}{2mc^2} = 7.38 \times 10^{-6}$ thus the frequency of the emitted photon is not modified much $\nu = \nu_0(0.99999261)$ where $\nu_0 = 2.92 \times 10^{15}$ is the frequency of the photon when the recoil energy is not accounted for. Thus the new frequency is $\nu = 2.919928421$. The corrected wavelength is $\lambda = 1.02741 \times 10^{-7}$m and the original wavelength is $\lambda_0 = 1.02739 \times 10^{-7}$m. Because the difference is so small the recoil energy is often neglected.