Nuclei

- Two particles outside closed shells
- Different shells only Clebsch-Gordan constraint
- Uncoupled states in the same shell \( |\Phi_{jm,jm'}\rangle = a_{jm}^\dagger a_{jm'}^\dagger |\Phi_0\rangle \)
- Note restriction due to Pauli principle: \( J=2j \) forbidden!
- Coupling
\[
|\Phi_{jj,JM}\rangle = \sum_{mm'} (j \ m \ j \ m' \ |J \ M) |\Phi_{jm,jm'}\rangle = \sum_{mm'} (j \ m' \ j \ m \ |J \ M) |\Phi_{jm',jm}\rangle \\
= \sum_{mm'} (-1)^{2j-J} (j \ m \ j \ m' \ |J \ M) (-1) |\Phi_{jm,jm'}\rangle \\
= (-1)^J \sum_{mm'} (j \ m \ j \ m' \ |J \ M) |\Phi_{jm,jm'}\rangle \\
= (-1)^J |\Phi_{jj,JM}\rangle
\]
- Only even total angular momentum
- With isospin
\[
|\Phi_{jj,JM,TMT}\rangle = \sum_{mm'm_{\frac{t}{2}}m'_{\frac{t}{2}}} (j \ m \ j \ m' \ |J \ M) (\frac{1}{2} m_t \frac{1}{2} m'_t \ |T \ M_T) |\Phi_{jmm_t,jm'm'_t}\rangle \\
= (-1)^{J+T+1} |\Phi_{jj,JM,TMT}\rangle
\]
- \( J+T \) odd!
$^{40}\text{Ca} + \text{two nucleons}$

- Spectrum
- $T=1 \ 0^+$ in $^{42}\text{Sc}$ below $T=0$ states due to “pairing” effect
- Most nuclei: ground state lowest possible total isospin
Orbits around $^{208}$Pb

- First empty proton level $h_{9/2}$ for neutrons $\rightarrow g_{9/2}$
Other examples

• 2 protons outside $^{208}\text{Pb}$
• $(h_{9/2})^2 \rightarrow J = 0, 2, 4, 6, 8$
• parity +
• 2 neutrons
• \((g_{9/2})^2\)
• \(J=0,2,4,6,8\)
• parity +

More

or p and n

\((\pi h_{9/2} \nu g_{9/2})\)

\(J = 0,1,2,...,9\)

parity -

\((\pi h_{9/2} \nu g_{9/2})\)

10^-?
More still

- $^{16}O$ plus 2 neutrons

$(\nu d_{5/2} \nu s_{1/2})^2$
and

- But in $^{18}$F
- T=0 ground state
- with J = 1
- Note J = 3 and 5
New case

- Excited states in $^{208}$Pb
How about $^{16}O$?

- Isospin coupling $\rightarrow T = 0$ or $1$

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**Oxygen-16**

Excitation Energy [MeV]

- $\pi^+$ $T = 0$ $\pi^-$ $T = 1$ $A \pm 1$ data
Improving excitation spectra beyond RPA

**Coupling of two-phonons in \(^{16}\text{O}\)**


Roughly:
- 80% \((0^+)\times(1^-) + 20\% \text{ ph}
- 98\% (3^-)^2
- 85\% (0^+)^2 + 15\% \text{ ph}

Need to do better:
- \(pp(hh)\) interactions
- 4-phonon states
[ Feshbach & Iachello (‘73)]
Yet another example

- $^{19}_0$
- 3 neutrons