Definition of magnetic moment

- Magnetic moment of a quantum state
  \[ \mu = \langle JM_J = J | \hat{\mu}_z | JM_J = J \rangle \]

- First quantization: magnetic moment operator
  \[ \mu^A = \sum_{i=1}^{A} \{ g_\ell(i) \ell_i + g_s(i) s_i \} \mu_N \quad \text{with} \quad g_\ell = \begin{cases} 1 & \text{proton} \\ 0 & \text{neutron} \end{cases} \]
  \[ g_s = \begin{cases} 5.58 & \text{proton} \\ -3.82 & \text{neutron} \end{cases} \]

- Nuclear magneton
  \[ \mu_N = \frac{e\hbar}{2M_p} \]

- Angular momentum operators "dimensionless"

- Second quantization
  \[ \hat{\mu}_z = \sum_{\alpha\beta} \langle \alpha | \mu_z | \beta \rangle a^\dagger_\alpha a_\beta \]
  \[ \text{With} \quad |\alpha\rangle = |n(\ell s = \pm) jm_j\rangle \]
  \[ \text{Operator can connect different spin-orbit partners!} \]

- See later and for M1 transitions
  \[ \mathcal{M}(M1, \nu) = \left( \frac{3}{4\pi} \right)^{1/2} \hat{\mu}_\nu \]

Evaluate single-particle magnetic moment

- Magnetic moment for single-particle state independent of radial wave function so
  \[ \mu_{j_2} = \langle (\ell \pm) jm_j = j | \mu_z | (\ell \pm) jm_j = j \rangle \]
  \[ = \frac{1}{j(j+1)} \langle (\ell \pm) jm_j = j | (\mu \cdot \ell + g_s s) \cdot j | (\ell \pm) jm_j = j \rangle \quad \text{Projection theorem} \]
  \[ = \frac{\mu_N}{j(j+1)} \langle (\ell \pm) jm_j = j | (g_\ell \ell + g_s s) \cdot j | (\ell \pm) jm_j = j \rangle \]
  \[ = \frac{\mu_N}{j(j+1)} \langle (\ell \pm) jm_j = j | (g_\ell \ell + g_s s) \cdot j | (\ell \pm) jm_j = j \rangle \]
  \[ = \mu_N g_\ell j + \frac{\mu_N}{2(j+1)} \langle (\ell \pm) jm_j = j | (g_s - g_\ell) s \cdot j | (\ell \pm) jm_j = j \rangle \]
  \[ = \mu_N g_\ell j + \frac{\mu_N}{2(j+1)} (g_s - g_\ell)(j^2 + s^2 - \ell^2) |(\ell \pm) jm_j = j \rangle \]
  \[ = \mu_N g_\ell j + \frac{\mu_N}{2(j+1)} (g_s - g_\ell)(j_2(j+1) + \frac{3}{2} - \ell(\ell + 1)) \]
  \[ = j\mu_N \left\{ g_\ell \pm \frac{g_s - g_\ell}{2\ell + 1} \right\} \]
  \[ \text{for} \quad j = \ell \pm \frac{1}{2} \quad \text{note “conserved” part of magnetic moment} \]
### Examples

- **Single particle or hole outside closed shell (IPM)**
- **Magnetic moment due to particle or hole since closed shell doesn't contribute**

**Particles**

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Spin</th>
<th>Magnetic Moment $\mu$</th>
<th>Experimental Magnetic Moment $\mu_{exp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{17}$F $\Rightarrow d_{5/2}$</td>
<td>$\frac{5}{2}$</td>
<td>$\frac{5}{2}(1 + \frac{4.58}{5})\mu_N = 4.79\mu_N$</td>
<td>$4.72\mu_N$</td>
</tr>
<tr>
<td>$^{17}$O $\Rightarrow d_{5/2}$</td>
<td>$\frac{5}{2}$</td>
<td>$\frac{5}{2}(0 - \frac{3.82}{5})\mu_N = -1.91\mu_N$</td>
<td>$-1.89\mu_N$</td>
</tr>
<tr>
<td>$^{41}$Ca $\Rightarrow f_{7/2}$</td>
<td>$\frac{7}{2}$</td>
<td>$\frac{7}{2}(0 - \frac{3.82}{7})\mu_N = -1.91\mu_N$</td>
<td>$-1.59\mu_N$</td>
</tr>
<tr>
<td>$^{209}$Bi $\Rightarrow h_{9/2}$</td>
<td>$\frac{9}{2}$</td>
<td>$\frac{9}{2}(1 - \frac{4.58}{11})\mu_N = 2.62\mu_N$</td>
<td>$4.08\mu_N$</td>
</tr>
</tbody>
</table>

**Holes**

<table>
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<tr>
<th>Nucleus</th>
<th>Spin</th>
<th>Magnetic Moment $\mu$</th>
<th>Experimental Magnetic Moment $\mu_{exp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{15}$N $\Rightarrow p_{1/2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}(1 - \frac{4.58}{3})\mu_N = -0.26\mu_N$</td>
<td>$-0.28\mu_N$</td>
</tr>
<tr>
<td>$^{15}$O $\Rightarrow p_{1/2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}(0 - \frac{-3.82}{3})\mu_N = 0.64\mu_N$</td>
<td>$0.72\mu_N$</td>
</tr>
<tr>
<td>$^{39}$K $\Rightarrow d_{3/2}$</td>
<td>$\frac{3}{2}$</td>
<td>$\frac{3}{2}(1 - \frac{4.58}{5})\mu_N = 0.13\mu_N$</td>
<td>$0.39\mu_N$</td>
</tr>
<tr>
<td>$^{207}$Pb $\Rightarrow p_{1/2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}(0 - \frac{-3.82}{3})\mu_N = 0.64\mu_N$</td>
<td>$0.59\mu_N$</td>
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</tbody>
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### Schmidt lines

- **Odd proton nuclei**
- **Lines: simple sp estimate**

![Schmidt Lines Diagram](image-url)
• Odd neutron nuclei

More nuclei
Odd-neutron nuclei

Nuclear Reactions

- Kinematic relations either in lab or COM frame (COM at rest)
- $X(a,b)Y$ reaction
Scattering angles in different frames

- From figure

\[ \tan \theta_{\text{lab}} = \frac{v_b' \sin \theta_{\text{COM}}}{v_{\text{COM}} + v_b' \cos \theta_{\text{COM}}} \]

- Also

\[ P = Mv_{\text{COM}} \]
\[ = (M'_a + M'_X)v_{\text{COM}} = M'_a v_a \]
\[ M'_a v'_a = -M'_X v'_X \]
\[ v_a = v'_a + v_{\text{COM}} \]

- Leads to

\[ v_{\text{COM}} = -v'_X = \frac{M'_a}{M'_X} v'_a \]

divide by \( v'_a \) and insert above for elastic process

\[ \tan \theta_{\text{lab}} = \frac{\sin \theta_{\text{COM}}}{\frac{M'_a}{M'_X} + \cos \theta_{\text{COM}}} \simeq \tan \theta_{\text{COM}} \quad M'_a \ll M'_X \]

Q-value

- For Q-values lab quantities are used for kinetic energies

- But can be expressed in terms of masses too so it is invariant as illustrated by

\[ Q = T_b \left( 1 + \frac{M'_b}{M'_Y} \right) - T_a \left( 1 - \frac{M_a}{M'_Y} \right) - 2(M'_a M'_b T_a T_b)^{1/2} \frac{M'_b}{M'_Y} \cdot \cos \theta \]

- unknown Q from two kinetic energies and scattering angle
Remarks

• Solve for $\sqrt{T_b}$ as a function of other KE and angle

$$\sqrt{T_b} = r \pm \sqrt{r^2 + s}$$

$$r = \frac{\sqrt{M'_a M'_b T_a}}{M'_b + M'_Y} \cdot \cos \theta$$

$$s = \frac{M'_Y Q + T_a (M'_Y - M'_a)}{M'_b + M'_Y}$$

• Distinguish
  
  - $Q > 0$ for $M'_a < M'_Y$ always solution: example

  - $Q < 0$ not always possible: threshold

  $$T_a^{th} = -Q \frac{M'_b + M'_Y}{M'_b + M'_Y - M'_a} > |Q|$$