Optical Trapping

The catalyst which motivated the development of optical trapping is often attributed to James Clark Maxwell when he unveiled the concept of radiation pressure. Apart from the significant amount of power that can be extracted as solar energy, it is remarkable how much force is imparted by photons, emitted by Sun or produced by lasers and/or other artificial devices, as a function of radiation pressure. A simple calculation that demonstrates the force exerted by a 1W laser on a dielectric sphere (for historical reasons), with a radius equivalent to the wavelength of the emitted photons ($\lambda 0.5145\mu m$), is determined below:

$$F_{rad} = \frac{2qP}{c} = 6.6 \times 10^{-5} \text{dyn} = 6.6 \times 10^{-10} \text{N}$$

where $q$ is the fraction of light reflected back $\sim 0.1$. This is similar to the calculation performed in class such that the subsequent acceleration of the sphere, with a uniform density of $1000 \text{kg/m}^3$, is $1.2 \times 10^6 \text{m/s}^2$ or roughly $10^5$ times the acceleration of gravity. Thus there is a substantial amount of energy that can be derived from this process and many innovations beyond the scope of this paper, such as interstellar travel via solar winds, are currently being explored.

There are alternative explanations to illustrate the means by which light exerts a force on objects. In the realm of atomic physics, as described by Einstein, “an atom that absorbs a photon of energy, $h\nu$, will receive a momentum impulse,” $p_{in}$, equal to $h\nu/c$ “along the direction of the incoming photon;” conversely, “if the atom emits a photon with momentum, $p_{out}$, the atom will recoil in the opposite direction” and the net change in momentum for the two events is given by $\Delta p = p_{in} - p_{out}$ for incoherent scattering. As
scattering is unpredictable, the resulting force is due to the absorbed photons and is directly proportional to the “number of photons scattered per second”, $N$:

$$F_{\text{scat}} = N \cdot p_{\text{int}}$$  \hspace{1cm} (2)

Coherent scattering of photons is based on the property of lensing in which the momentums of light are altered according to Newton’s Third law whereby the lens experiences a reaction force, or equivalently, the dipole force since electric fields induce a dipole moment, $p$, on the particle. When the “dipole moment is in phase with $E$, the interaction energy, $-p \cdot E$, is lower in high field regions”; if the quantities are out of phase, there is an increase in the amount of energy of the particle and it is subjected to a force that is equal and opposite to the rate $\Delta p$ consequently pushing it out of the field. The result is that a positive lens is drawn to region of high intensity and an upward force is exerted on a glass bead or particle in order to balance the scattering force.

Pioneering the field of optical trapping was Arthur Ashkin who had shown in 1970 using glass beads, which are far more polarizable than atoms, that it was possible to trap micron-sized particles with a pair of focused beams of laser light. When the force gradients, created by the alternating electromagnetic field of light, cause a particle to move, the effect is termed photophoresis; the magnitude of these forces significantly dwarf that of radiation pressure. We predominately look at the realm of ray (Mie) optics ($d > \lambda$) as opposed to the Rayleigh regime ($d < \lambda$) where particles can be considered dipoles. Experimenting with latex spheres, whose surfaces act as focusing lenses, of sizes 0.59-, 1.31-, and 2.68$\mu$m freely suspended in water, Ashkin observed that off-centered spheres were simultaneously drawn into the beam axis of a several milliwatt laser and accelerated in the direction of the incoming light. When the laser is blocked the sphere will wander along the fringes of the beam due to Brownian motion. Furthermore, it has
been experimentally verified that through the use of more powerful lasers it is possible to solely influence larger-sized spheres and leave smaller ones behind. Although the path of light through a refractive medium such as a glass sphere can be conveniently deduced according to Snell’s Law:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]  

(3)

it is important to understand how the forces at work are derived from the momentum changes of the incident beam. The following diagram depicts a glass sphere displaced from the beam axis such that two incident rays of varying intensity, a and b, undergo Fresnel reflection and refraction (deflection):

![Diagram](image)

The stronger ray, a, is internally refracted (and reflected) resulting in radiation pressure forces, \( F_D^i \) and \( F_D^o \) (input and output deflection) and \( F_R^i \) and \( F_R^o \) (input and output reflection) which convey the motion of the particle in the direction opposing the incident light. All forces denote acceleration in +z direction with the radial reflection forces principally canceling with one another while the radial deflection forces add in the –r direction (towards axis); since the rays closer to the beam are of greater intensity, the
added radial force from (a) rays overwhelm the opposing forces due to (b) rays. In summary, there is a transverse component to the force defined as the gradient force, $F_{\text{grad}}$, that draws the particles into the axis and a scattering force, $F_{\text{scat}}$, which pushes the sphere forward within the beam. The gradient force of an electromagnetic wave on a neutral atom (simple dipole) is given by the dipole force formula:

$$F_{\text{grad}} = \frac{1}{2} \alpha \nabla E^2$$

(4)

where $\alpha$ is the optically induced polarizability of the particle. It is possible to compute the effective velocity of the sphere as a consequence of the laser by applying Stoke’s Law (for $r \ll w_0$) with the following parameters: ($q = 0.06$, $r_{\text{sphere}} = 1.34\mu m$, $\eta_{\text{H}_2\text{O}} = 1 \times 10^{-2}\text{P}$, $w_0 = 6.2\mu m$, and $P = 19\text{mW}$)

$$v = \frac{2qPr}{3c\pi w_0^2\eta}$$

(5)

where $w_0$ represents the width of the Gaussian beam.

It naturally occurred to Ashkin that two opposing Gaussian beams will force a particle into stable equilibrium in which any displacement yields a restoring force and a true optical potential well via radiation pressure alone is achieved. The viscous media used provides a damping effect that dissipates all thermal energies normally acquired from trapping potentials; these quantum effects will be briefly explored later in this paper during our discussion of ‘optical molasses.’ For a better view of how the radiation pressures arising from reflection and refraction are consolidated into the scattering and gradient forces, refer to the figures below which also illustrate several other experiments including the levitation trap in which a laser directed upwards will induce a scattering force that perfectly cancels with that of gravity leaving a particle suspended in midair:
Optical tweezers utilize a focused Gaussian beam that confines the particle due to the dominance of a backward gradient force over the forward scattering force; this is achieved since a light gradient is formed near either focus, whose sharpness improves the degree of the gradient and is optimal when the diameter of the sphere is approximately the wavelength of the laser light. A displacement towards either of the focal points of the light results in an imbalance of scattered light that pushes the particle to the center of the trap. Auxiliary cooling beams are also used to maintain an equilibrium temperature at thermal energies. Hence, the outwardly directed scattering force, along the radial
direction, can be overcome by the attractive dipole force altogether dictating the motion of the particle to one of the foci:

\[
F = Q(n_1P/c) 
\]

where \(n_1\) is the incident momentum per second in a medium of index of refraction, \(n_1\), for a ray of power, \(P\). As in the case of reflection, the maximum force due to radiation pressure corresponds to \(Q = 2\); at \(n = 1.2\), the maximum gradient force of \(Q_{\text{max}} \sim 0.5\) illustrated below:
It is curious to note that a critical angle is reached when $\theta = 80^\circ$ at which point the gradient force no longer wins over the scattering force. For a more quantitative measurement, we shall consider the force on a bead from a 10mW laser as defined in class and analogous to equation 2, where $N$ is the number of photons striking the sphere per second and equal to $P/(hc/\lambda)$:

$$F = \Delta p^*N = \frac{(h/\lambda)(P/(hc/\lambda))}{(3\times10^8 \text{m/s})} \approx 3 \times 10^{-11} \text{N} \approx 30 \text{pN}$$

One should observe that the force is proportional to the power of the laser and a subsequent increase in wattage should theoretical produce a force several orders of magnitude greater; this is presumably why many consider optical tweezers to be the microscopic version of the science-fictional tractor beam. However, the value of $N$ is only an estimate and distorted since many complex processes of absorption and isotropic scattering of photons involving application of the Einstein coefficients for stimulated emission are involved.
To successfully trap atoms in a three-dimensional environment, Steven Chu, at Bell Labs, believed that despite the little trapping volume offered by a single beam it was possible to confine atoms undergoing many random walks in a potential well if they were sufficiently slowed down. To overcome thermal effects which dictate that the average atomic energies are $\frac{3}{2}k_bT \sim \frac{1}{2}mv^2$ and substantially greater than trapping forces could contain, they were able to establish an array of laser beams that collectively produced a net force opposing the particle’s motion such that atoms moving slow enough were found to be experiencing a force analogous to viscous damping:

$$F = -\gamma v$$

(10)

These light ‘cooled’ atoms were now akin to pollen particles in water experiencing Brownian motion where the drag force felt in time, is given by the random walk equation:

$$\langle x^2 \rangle = 2Dt$$

(11)

where the diffusion constant, according to the Einstein relation, is

$$D = \frac{k_bT}{\gamma}$$

(12)

Surrounding atoms with 6 laser beams in +/- x, y and z directions created a ‘sea of photons’ which acts as a very viscous fluid dubbed ‘optical molasses’:
There is extensive atomic physics associated with this optical trap including Doppler shifts of the absorption spectrum that create the opposing forces described above as well as quantum effects which give rise to heating processes that must be explicitly counter-balanced by cooling rates to ensure equilibrium temperatures. Once atoms could be slowed down to manageable velocities on the order of 250 cm/s (almost stopped) respectable confinements lasting for a second and longer, were observed within an area compressed into 2 mm diameter spaces. The first three-dimensional stable atom trap was achieved.

In order to utilize laser tweezers, it is usually necessary to develop detection techniques such as interferometry in which, as illustrated in the diagram below, two beams of orthogonal polarization are recombined after hitting their target and whose ensuing intensities can be analyzed so as to determine the position of the sphere in the trap:
The restoring force that continues to maintain the particle’s position can be modeled as a spring where the force is proportional to the distance between the sphere and the focus and a stiffness constant, $k$. Some recent experiments that involved this approach include twisting bacterium about a tethered flagellum thereby elucidating more about the structure and mechanism of the bacterial motor as well as numerous other molecular motors such as kinesin and RNA polymerase. To study the latter molecule, scientists typically attach it and elongated DNA to a glass micropipette and a polystyrene bead held in an optical trap (see Figure 7). The stretching of DNA by displacing the micropipette generates a force that opposes the work of transcription and will continue to do so prior to a particular value is observed or has been reached, whereby the forces of the motor and those required to stall it are equivalent and are on the order of 25pN.
Additionally, on examination of the driving force of spherical mitochondria in cells, scientists were able to isolate single molecules by applying optical tweezers at a maximum power setting of 220mW. This excessive power is required to generate a trapping force that is greater than the driving force due to multiple motors that guide the molecule along microtubules. Pictured in figure 8 are several screenshots taken at varying time scales where laser tweezers are switched from on to off so as to illustrate a mitochondrion is approaching, eventually trapped and subsequently released from the trap:

![Figure 8](image)

Once ‘captured’ a measure of the driving force can be derived upon lowering the power of the optical trap until the mitochondrion is able to escape with relatively no change in its usual speed. The force was measured to be about 150mW which corresponds to an average driving force of 63mW per motor on mitochondria. Since these molecules are moving at fast velocities within their regime, it is not only difficult to trap them but obtain conclusive results regarding their molecular forces. The optical trap’s effectiveness is based primarily on the mitochondria’s absolute size and given index of refraction; it is possible, once again, to apply Stoke’s law to the escape velocities of mitochondria subject to viscous drag forces. Considering the driving force of one motor (63mW), the corresponding force associated with a single motor molecule is approximately 2.6pN. Moreover, the constant velocity or stepping rate of these motors
leads us to presume that they are able to maintain these fixed rates over a wide range of ever increasing viscous drag much like the stepping actions of kinesin molecules along microtubules.

The ability to manipulate molecules on such a fine scale with precision accuracy has broadened the scope of experimental research in nearly all fields of science. With improved technology and many new tests being developed we are able to further explore this now tangible world of atomic physics and of the cell. Given the capabilities photons offer us, the old adage may seem appropriate: “One day, Sir, you may tax [them].”

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1 Michael Faraday’s reply in the 1850’s to a question posed by the British Minister of Finance regarding whether electricity had any practical value.
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