Problem Set 8 (50 points)

Suggested Reading: Harris: 10.4-10.7, Griffiths 6.1, Harris: Guidelines for computational exercises, page 193)

1. Harris 10.50 (10 points)

2. Perturbation theory (20 points): As we discussed in class, we can write a perturbed Hamiltonian as \( H = H_0 + H' \). If we know the energy eigenstates \( \psi_n \) of \( H_0 \) such that \( \psi_n H_0 = E_n \psi_n \), then the first order correction to the energy of eigenstate \( \psi_n \) is given by

\[
E_n^{(1)} = \langle \psi_n | H' | \psi_n \rangle
\]

a) Using this result, calculate the first order correction to the allowed energies of an infinite square well (of width \( L \)) which has a delta-function bump in the middle of the well:

\[
H'(x) = \alpha \delta(x - L/2),
\]

where \( \alpha \) is a constant that determines the strength of the bump. Comment on why the energies are not perturbed for even \( n \).

b) Now consider the perturbation,

\[
H'(x) = \begin{cases} 
0 & \text{for } x \leq L/2 \\
U_0 & \text{for } L/2 < x < L 
\end{cases}
\]

That is, half of the infinite square well has the floor raised by an amount \( U_0/2 \). What is the first order perturbation to the energies?

3. Harris 10.80: mathematica! (20 points) Note, you’ll adapt the code you used on the previous computational problems and follow along similar lines. Start by plotting the potential function to make sure that looks as you would expect. If you find it laborious to find the 14 allowed energies by hand don’t complain to me... Write a program to find them for you! (although this surely will take much longer) If you are interested in this approach, you can nest another for-loop around the one that solves the Schrödinger equation and look for the sign change of the tail of the wavefunction. You are welcome to make your scatter plots in a different program if you prefer, though it is not difficult to do in mathematica. (15 points)