In class work

1. Consider the “two sides of the box” problem with \( N \) distinguishable particles.
   (a) What is the multiplicity \( \Omega \) for all the particles to be on one side? What is the multiplicity for half of the particles to be on one side?

(b) Use Stirling’s formula \( N! = \sqrt{2\pi N} e^{-N} N^N \) to simplify your answer for part (a). Calculate the entropy the system \( S = k \ln \Omega \) when all the particles are on one side and when half of the particles are on one side.

(c) Note that in part (b) the volume of the system effectively doubles when we compare all the particles on one side of the box versus equally distributed. Thus, we should get the same change in entropy from the thermodynamic relation \( dS = \frac{dQ}{T} \) in the process of reversible isothermal expansion that doubles the volume of the gas. Calculate the change in entropy in this case (we did this in class) and compare to the answer in (b). Do they agree?
2. Paramagnet. (a) A simple model of a paramagnet consists of \( N \) spin-1/2 electrons localized on different atoms. In an external magnetic field \( B \) each electron has energy \( \pm \mu B \) where \( \mu \) is the magnetic moment and is constant. The + sign corresponds to spin up. Calculate the energy \( E \) of a system where \( p \) spins are up and \( N - p \) spins are down. What is the maximum energy? What is the minimum energy?

(b) Given that the number of ways to have \( p \) spins up and \( N - p \) spins down is \( \frac{N!}{p!(N-p)!} \), what is the entropy \( S(N,p) \) for this system? Since we are dealing with logarithms, it makes sense to use this version of Stirling’s approximation: \( \ln(N!) = N\ln N - N \) (Explain why this is in agreement with Stirling’s formula.)

(c) What value of \( p \) maximizes the entropy? What is this energy?

(d) Make a rough sketch of entropy versus energy of the system (or use mathematica to plot it). If we imagine adding heat to the system without doing any work, then the change in the total energy \( dE = dQ \), we can thus re-arrange the second law to be,

\[
\frac{dS}{dE} = \frac{1}{T}
\]

From the sketch of \( S \) versus \( E \), comment on where the temperature is positive and negative. Which is “hotter”... positive or negative temperature?