In class work

Questions for discussion

1. Is the exchange force a real force?

2. If two electrons do not have overlapping wave functions, the exchange force effect disappears. Explain this in the context of distinguishability.

3. What role does spin play in the discussion of bonding/anti-bonding orbitals in the hydrogen molecule?

4. Here we consider adding electrons to two finite wells (a simple model for two atomic nuclei) that are either close together or far apart. The figure shows the lowest energy eigenstates for the two cases.

(a) Discuss why the energy eigenstates are as shown in the drawing. (b) For the situation on the left, to yield the lowest energy the first electron would occupy the state $\psi_A$ and would be shared between both wells. The second electron would occupy that state with opposite spin. A third electron would occupy state $\psi_B$. Now consider the two wells far apart (on the right). In this case, the wave functions for each well do not overlap and wave functions $\psi_A$ and $\psi_B$ have nearly equal energy. $\psi_A$ and $\psi_B$ have nearly identical shapes in the right well and only have opposite sign on the left. Because they are of equal energy, we might as well as take sums or differences of $\psi_A$ and $\psi_B$, so an electron can occupy the state $\psi_A$, $\psi_B$, $\psi_A + \psi_B$, or $\psi_A - \psi_B$. Explain why in this spread-out situation, electrons can be put in one well without violating the exclusion principle - no matter what states are occupied in the other well.

(c) Write the total wavefunction of the two electron states for each case.
5. Three particles (from the homework) : Suppose that you had three particles, one in state $\psi_a(x)$, one in state $\psi_b(x)$, and one in state $\psi_c(x)$. Assuming that $\psi_a, \psi_b$ and $\psi_c$ are orthonormal, construct three particle states representing (a) distinguishable particles, (b) identical bosons, and (c) identical fermions. (Keep in mind that (b) must be completely symmetric under interchange of any pair of particles and (c) must be completely anti-symmetric in the same sense.) Note: there is a handy trick for constructing completely anti-symmetric wave functions: Form the “Slater determinant”, that is, take the determinant of a matrix where the first row is $\psi_a(x_1), \psi_b(x_1), \psi_c(x_1)$ and the second row is $\psi_a(x_2), \psi_b(x_2), \psi_c(x_2)$ etc.

(d) Explain what property of the determinant ensures that switching the labels on any two particles will change the sign of the multiple particle state.